Asymmetric Coupled-Transmission-Line Magic-T

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Summary—A Magic-T was designed using a 3-db asymmetric coupled-transmission-line directional coupler and a Schiffman phase shift network. Expressions for the coupling and phase difference of the outputs of the basic coupler were obtained. Two phase shift networks for compensating the phase variation in the coupled outputs were investigated. The Schiffman phase shift network was shown to provide the best compensation over a given bandwidth. One unit designed in strip transmission line to operate from 2 to 4 Gc showed good agreement with the theory. The maximum difference in output power was 0.4 db, and the phase deviation was less than 4°. The isolation between the input ports was greater than 25 db, while the isolation between output ports was greater than 21 db throughout the entire band. Levy's design equations for a two-section asymmetric coupler with optimum coupling distribution are given for completeness.

Introduction

A VARIETY of Magic-T's have been developed over the years. Two of the better known ones are the ring hybrid and the Alford Coaxial Magic-T.[1] However, there still appears to be a need for a printed-circuit broad-band Magic-T that can be incorporated into completely integrated strip transmission-line designs.

This paper describes a Magic-T design using two relatively new transmission-line circuits: a two-section 3-db asymmetric coupled-transmission-line directional coupler,[2] and a Schiffman phase shift network.[3] The 3-db asymmetric coupled directional coupler can be designed to split the input power equally between the two outputs over a wide frequency band. The phase difference between the outputs, unlike that of a symmetric coupler, is not inherently 90°. Depending on the input port used, the outputs are either in phase or 180° out of phase at midband, the phase difference being a function of frequency. The variation of phase with frequency can be compensated for with a phase shift network. The combination of these two circuits results in an octave bandwidth four-port device that has the properties of a classical Magic-T.

Coupler Analysis

The operation of the Magic-T shown in Fig. 1(a) will be explained from an analysis of the two-section asymmetric coupler and the phase shift network. An expression for the phase difference between the outputs of the coupler will be determined. It will be shown that for the combined network port 1 couples to ports 2 and 4 in phase, and that port 3 couples to ports 2 and 4 in antiphase.

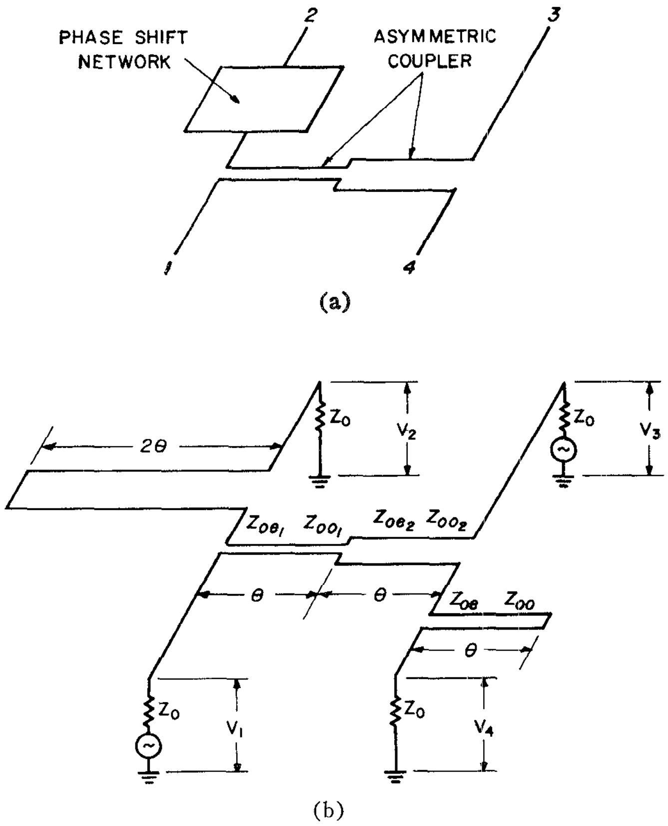

Fig. 1—Schematic diagram of Magic-T. (a) Basic configuration. (b) Actual design.

The general case of an n-section asymmetric coupled-transmission-line directional coupler has been analyzed in a paper by Levy.[2] He has shown that a multi-element directional coupler is equivalent to a stepped-impedance transmission-line filter under the condition that

$$Z_{oe_1}Z_{oo_1} = Z_{oe_2}Z_{oo_2} = \cdots = Z_{oe_n}Z_{oo_n} = 1 \qquad (1)$$

where Z_{oe} and Z_{oo} are the normalized even- and odd-mode impedances of a coupled section. The coupling from port 1 to 4 and from port 1 to 2 is equal to the transfer and reflection coefficients, respectively, of a cascaded set of transmission lines of electrical length θ with the characteristic impedances $Z_{oe_1}, Z_{oe_2}, \cdots, Z_{oe_n}$.

Therefore, the coupler can be analyzed by use of the $ABCD$ matrix of an n-section cascaded two-port net-

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The author is with Loral Electronics Corporation, New York, N. Y.
[1] A. Alford and C. W. Watts, "A wide-band coaxial hybrid," 1956 IRE NATIONAL CONVENTION RECORD, Pt. 1, pp. 171–179.
[2] R. Levy, "General synthesis of asymmetric multi-element coupled transmission-line directional couplers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-11, pp. 226–237; July, 1963.
[3] B. M. Schiffman, "A new class of broad-band microwave 90-degree phase shifters," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 232–237; April, 1958.

work. The reflection and transfer coefficients of a two-port network are given by the familiar formulas

$$\Gamma = V_2/V_1 = \frac{(A - D) + j(B - C)}{(A + D) + j(B + C)} \tag{2}$$

$$T = V_4/V_1 = \frac{2}{(A + D) + j(B + C)} \tag{3}$$

and

$$V_2/V_4 = \frac{(A - D) + j(B - C)}{2} \tag{4}$$

where Γ and T are the reflection and transfer coefficients respectively, of the two-port network. The voltages are the terminal voltages of the four-port coupler. The phase difference between V_2 and V_4 is given by

$$\phi = \theta_4 - \theta_2 = \tan^{-1}\left[\frac{B - C}{A - D}\right] \tag{5}$$

where the sign of the phase is defined by

$$\frac{V_2}{V_4} = \frac{|V_2| e^{-j\theta_2}}{|V_4| e^{-j\theta_4}} = \left|\frac{V_2}{V_4}\right| e^{-j\phi}. \tag{6}$$

An important distinction between a symmetric and an asymmetric coupler can be seen from a property of two-port networks. If the elements of the $ABCD$ matrix of a two-port network turned around are defined as $A'B'C'D'$ with the usual conventions of current maintained, the following relationship exists:

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} D & B \\ C & A \end{bmatrix}. \tag{7}$$

For a symmetric coupler where $A = D$, this means that the phase difference of the outputs is independent of which port is chosen as the input port. However, for an asymmetric coupler, it can be seen from (4) and (5) that the phase difference between the outputs depends on the choice of input ports. A change in the input port changes the output phase difference from ϕ to $\phi+\pi$ (*e.g.*, if the outputs for a signal in port 1 were in phase, the outputs of a signal from port 3 would be 180° out of phase).

For the two-section asymmetric coupler the $ABCD$ matrix is given by

$$\begin{bmatrix} A & jB \\ jC & D \end{bmatrix} = \begin{bmatrix} \cos^2\theta - \dfrac{Z_{oe_1}}{Z_{oe_2}}\sin^2\theta & j(Z_{oe_1} + Z_{oe_2})\sin\theta\cos\theta \\ j\left(\dfrac{1}{Z_{oe_1}} + \dfrac{1}{Z_{oe_2}}\right)\sin\theta\cos\theta & \cos^2\theta - \dfrac{Z_{oe_2}}{Z_{eo_1}}\sin^2\theta \end{bmatrix} \tag{8}$$

where Z_{oe_1} and Z_{oe_2} are the normalized even-mode impedances of the coupling sections. The phase difference obtained by substituting the above expressions into (5) is given by

$$\phi = \tan^{-1}(K_1 \cot\theta) + m\pi \tag{9}$$

where

$$K_1 = \frac{Z_{oe_1} + Z_{oe_2} - \left(\dfrac{1}{Z_{oe_1}} + \dfrac{1}{Z_{oe_2}}\right)}{\dfrac{Z_{oe_2}}{Z_{oe_1}} - \dfrac{Z_{oe_1}}{Z_{oe_2}}}. \tag{10}$$

In the range $0<\theta<\pi$ the values of ϕ are restricted to

$$-\pi/2 < \tan^{-1}(K_1 \cot\theta) < \pi/2 \tag{11}$$

and

$m = 0$ for K_1 positive; *i.e.*, $Z_{oe_2} > Z_{oe_1}$

$m = 1$ for K_1 negative; *i.e.*, $Z_{oe_2} < Z_{oe_1}$.

At midband,

$$\theta = \pi/2 \quad \text{and} \quad \phi = 0 \qquad \text{for } Z_{oe_2} > Z_{oe_1},$$

and

$$\phi = \pi \qquad \text{for } Z_{oe_2} < Z_{oe_1}$$

Phase Shift Network

An examination of (9) shows how the phase difference of the outputs of a two-section asymmetric coupler varies with frequency. If over a given bandwidth the outputs referred to input port 1 could be kept in phase, the outputs referred to input port 3 would be 180° out of phase. The network would then operate as a Magic-T over that bandwidth. The possibility of using a simple length of line or a more complicated phase shift network to maintain the outputs in phase will be considered.

If a length of line 2θ long is added to port 2, assuming K_1 negative, the phase difference between the outputs at the new reference planes is

$$\psi = \tan^{-1}(K_1 \cot\theta) + \pi - 2\theta. \tag{12}$$

The maximum or minimum values of ψ occur when

$$\frac{d\psi}{d\theta} = 0; \text{ for } \theta = \theta_m = \tan^{-1} \pm \sqrt{-K_1}\sqrt{\frac{1 + 2K_1}{K_1 + 2}}. \tag{13}$$

In order to determine if this length of line will provide sufficient phase compensation, an actual design will be worked out. The design procedure used to determine the impedances of the coupling sections of the two-section coupler is described in the Appendix. For a 3-db coupler with a 2:1 bandwidth, the following results were obtained:

$$\text{Coupling ripple} \pm 0.03 \text{ db}$$
$$\beta = 1.0053$$
$$h = 0.1446$$
$$Z_{oe_1} = 2.9651$$
$$Z_{oe_2} = 1.2328.$$

Using (1) for the odd- and even-mode impedances, and renormalizing to 50 Ω gives

$$Z_{oe_1} = 148.3 \ \Omega \qquad Z_{oo_1} = 16.9 \ \Omega$$
$$Z_{oe_2} = 61.6 \ \Omega \qquad Z_{oo_2} = 40.6 \ \Omega$$

and from (10),

$$K_1 = -1.532.$$

For a length of line 2θ added to arm 2, it can be seen from (13) that for $0 > K > -2$, ψ is a monotonically increasing function of θ. Therefore, the maximum and minimum values of ψ occur at the band edges, where $\theta = 60°$ and $120°$. Substituting these values into (12) gives $\psi = \pm 18.6°$. This phase difference is normally not acceptable in a Magic-T.

Replacing the length of line with the phase shift network, it can be determined if the network will provide sufficient phase compensation in the above design. As shown in Fig. 1(b) the phase shift network consists of a length of line 4θ long added to port 2, and a coupling section θ long added to port 4. The phase difference at the outputs of these new reference planes is given by

$$\psi = \phi + \gamma - 4\theta = \tan^{-1}(K_1 \cot\theta) + \cos^{-1}\left[\frac{\rho - \tan^2\theta}{\rho + \tan^2\theta}\right] + \pi - 4\theta \tag{14}$$

here

$$\gamma = \cos^{-1}\left[\frac{\rho - \tan^2\theta}{\rho + \tan^2\theta}\right]^{(3)} \tag{15}$$

and $\rho = Z_{oe}/Z_{oo}$ where Z_{oe} and Z_{oo} are the even-mode and odd-mode impedances of the coupling section of the phase shift network.

The maximum and minimum values of ψ are not easily determined from (14). Therefore, the phase difference of the unit with a Schiffman phase shift network was determined graphically. A plot of ψ as a function of θ, for several values of ρ, is shown in Fig. 2. It can be seen that the minimum phase difference is obtained for $\rho = 1.8$. Over a 2:1 band the maximum deviation is 3°. This should be quite acceptable for most applications employing a Magic-T.

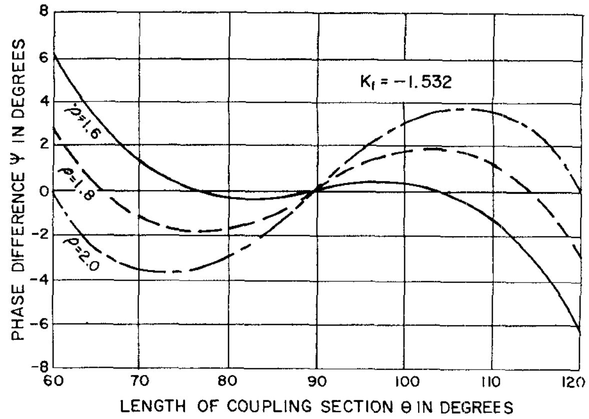

Fig. 2—Phase difference of asymmetric coupler with Schiffman phase shift network.

Experimental Data

An experimental Magic-T was designed using the above technique. Printed circuit construction was employed and the dimensions of the Magic-T, shown in Fig. 3, were calculated from Cohn's formulas.[4,5] The material used was 1 ounce copper clad polyethylene $\frac{1}{8}''$ thick with a dielectric constant of 2.32 ± 0.01. The design parameters for the Magic-T are shown in Table I.

TABLE I

	Section 1 (Ω)	Section 2 (Ω)	Phase Shift Network (Ω)
Z_{oe}	148.3	61.6	67.0
Z_{oo}	16.9	40.6	37.3

The section of the board requiring tight coupling was made as a broad side coupler. The 0.12 polyethylene shim shown in section A-A' of Fig. 3 only extends over the tightly coupled area. When the boards are clamped together there is a small air gap immediately adjacent to the shim which apparently does not affect the operation of the circuit. The loosely coupled section and the coupling section of the phase shift network were made as coplanar couplers.

The measured values of input VSWR into ports 1 and 3 are shown in Fig. 4, and are less than 1.4:1. The measured values of amplitude balance, isolation, and phase difference are shown in Fig. 5. The maximum difference in power between ports 2 and 4 is 0.4 db and the phase deviation is less than 4°. The isolation between ports 1 and 3 and ports 2 and 4 is greater than 25 db and 21 db respectively. The unit operates over an octave bandwidth and has good power division and high isolation.

[4] S. B. Cohn, "Characteristic impedance of broadside coupled strip transmission lines," IRE Trans. on Microwave Theory and Techniques vol. MTT-8, pp. 633–637; November, 1960.

[5] S. B. Cohn, "Shielded coupled-strip transmission lines," IRE Trans. on Microwave Theory and Techniques, vol. MTT-3, pp. 29–38; October, 1955.

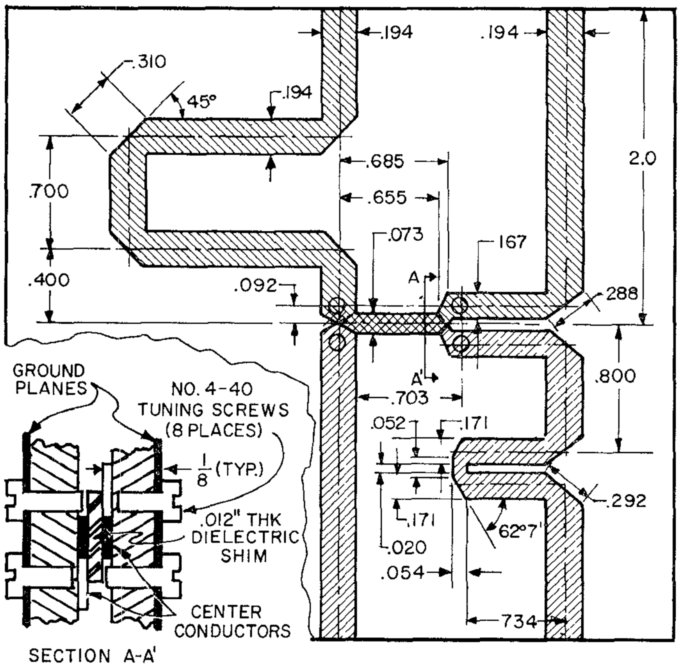

Fig. 3—Asymmetric coupled-transmission-line Magic-T.

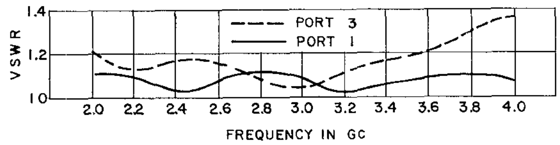

Fig. 4—Input VSWR of Magic-T.

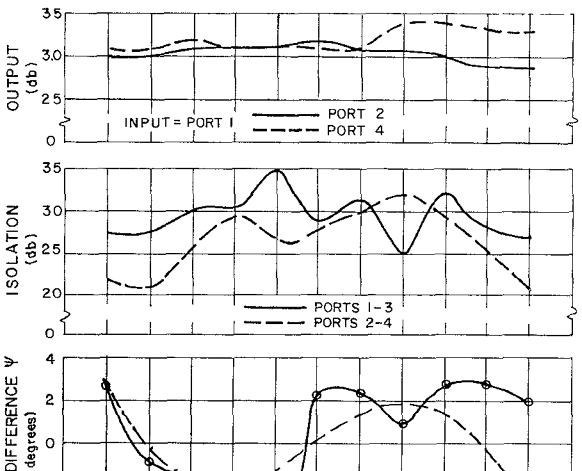

Fig. 5—Transmission loss and phase difference of Magic-T.

Practical Design Considerations

Several practical problems arise in the design of this type of Magic-T. The coupling of one section of the asymmetric coupler will always be considerably tighter than the over-all coupling. In the design of a 3-db coupler, this can necessitate the use of two different coupling configurations, complicating the junction between them. This junction, the phase shift network, and the junction between the output lines introduce internal reflections which account for most of the deviation of the unit from the calculated values.

The connection between lines of the coupled section of the phase shift network can cause high internal reflections. However, these reflections can be greatly minimized by careful design of the connection. The reflections from the other junctions cannot be sufficiently reduced by modifying the junctions.[6] Tuning screws, shown in Fig. 3, were used to reduce these effects and improved the over-all performance of the network. Further improvement in performance of the network might also be achieved with one of several other coupling configurations.[7,8]

Conclusions

The broad-band Magic-T was designed with a two-section asymmetric coupled-transmission-line directional coupler and a Schiffman phase shift network. The phase shift network compensates for the phase variation in the coupler. The resulting network is an octave bandwidth Magic-T with low VSWR, good power division, and high isolation.

The approach used represents an additional technique for the design of Magic-T's. The resulting network has several advantages over other types of hybrids. The transmission-line ring hybrid, while smaller and simpler to design, has a narrower bandwidth. The Alford Magic-T,[1] which provides higher isolation, is difficult to construct in strip transmission line at frequencies above the UHF region. The asymmetric coupled-transmission-line Magic-T lends itself to printed circuit techniques and should prove useful in the design of completely integrated strip transmission-line packages.

Appendix

Design Procedure for Two-Section Asymmetric Coupler

The main advantage of an asymmetric coupler is that it provides one more degree of freedom in selecting the coupling values of the different sections. Levy[2] has devised a synthesis procedure for n-section asymmetric couplers having the following Chebyshev distribution:

$$\frac{|S_{12}|^2}{|S_{11}|^2} = \beta^2 - h^2 T_n^2\left(\frac{\cos\theta}{\cos\theta_0}\right). \tag{16}$$

[6] G. L. Matthaei, S. B. Cohn, E. M. T. Jones, P. S. Contes, Jr., W. J. Getsinger, J. T. Belljahn, C. C. Flammer, J. K. Shmizer, and B. M. Schiffman, "Design Criteria for Microwave Filters and Coupling Structures," Stanford Research Institute, Menlo Park, Calif., S.R.J. Final report, Project 2326, Contract DA 36-039 SC 74862, p. 113; January, 1961.

[7] W. G. Getsinger, "A coupled stripline configuration using printed-circuit construction that allows very close coupling," IRE Trans. on Microwave Theory and Techniques, vol. MTT-9 pp. 535–544; November, 1961.

[8] W. J. Getsinger, "Coupled bars between parallel plates," IRE Trans. on Microwave Theory and Techniques, vol. MTT-10, pp. 65–72; January, 1962.

T_n denotes the Chebyshev function of the first kind of order n

S_{12}^2 = power transferred from port 1 to 2
S_{14}^2 = power transferred from port 1 to 4

where

$$\frac{1}{\cos\theta_0} = \cosh\left(\frac{1}{n}\cosh^{-1}\beta/h\right) \qquad (17)$$

and the pass band extends from θ_0 to $\pi-\theta_0$. Another relationship for the constants β and h can be obtained by applying the condition that the coupling ripple, in db, is to be symmetrical about the nominal coupling value. This condition gives

$$10\log_{10}\left[\frac{\beta^2-h^2}{1+\beta^2-h^2}\times\frac{\beta^2}{1+\beta^2}\right] = 2C \qquad (18)$$

where C is the nominal coupling value in db.

The results of this synthesis for a two-section asymmetric coupler will be repeated here.

$$Z_{oe_1} = \frac{b}{d+1} \qquad Z_{oe_2} = dZ_{oe_1}. \qquad (19)$$

Z_{oe_1} and Z_{oe_2} are the normalized even-mode impedances of the two-section coupler shown in Fig. 1(b). For a coupler with a specified coupling value and bandwidth, the values of β and h can be determined from (17) and (18). The constants b and d are defined in terms of β and h as follows:

$$b = \sqrt{(1+\beta^2)+(\beta+h)\sqrt{1+\beta^2}+\beta h} + \sqrt{(1+\beta^2)-(\beta+h)\sqrt{1+\beta^2}+\beta h} + \sqrt{2\beta(\beta+h)} \qquad (20)$$

$$d = \sqrt{1+\beta^2-h^2} - \sqrt{\beta^2-h^2}. \qquad (21)$$

The normalized odd-mode impedances can be obtained from the relationship

$$Z_{oe_1}Z_{oo_1} = Z_{oe_2}Z_{oo_2} = 1. \qquad (22)$$

It has been shown by several investigators that (22) represents the condition for which, independent of frequency, the coupler is matched and has infinite directivity.[9,10]

Acknowledgment

The author wishes to thank L. Teichman for his significant help in performing the measurements.

[9] E. M. T. Jones and J. T. Belljahn, "Coupled-strip-transmission-line filters and directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 75–81; April, 1956.
[10] J. K. Shimzer and E. M. T. Jones, ""Coupled-transmission-line directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 403–410; October, 1958.

Synthesis of Filter-Limiters Using Ferrimagnetic Resonators

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Summary—**A synthesis procedure is developed for microwave band-pass filters with the Chebyshev response using orthogonal circuit resonators coupled by a ferrimagnetic resonator. A stripline ferrimagnetic resonator filter is analyzed in detail. Equations and graphs are given which allow the selection of ferrimagnetic material and size of the ferrimagnetic sample necessary to achieve a desired bandwidth and insertion loss for a given pass-band response. The theoretical behavior of these circuits as microwave power limiters is discussed and it is shown that the ratio of the limiting threshold to the filter bandwidth is a constant depending only on the pass band response shape. Experimental confirmation of the design information is discussed as well as some practical methods of varying the limiting threshold.**

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I. Introduction

The COUPLING of small ferrimagnetic ellipsoids to microwave transmission lines has been discussed by a number of authors [1]–[3]. It is found that when the ferrimagnet is excited in the uniform precession it behaves like a lumped constant resonator with, in certain cases, a nonreciprocal phase behavior. The resonant frequencies of such a resonator can be tuned with a dc magnetic field. Its unloaded $Q(Q_u)$ depends on the surface polish of the sample and doping with impurity ions. For example, with highly polished yttrium iron garnet (YIG) spheres Q_u can be as great as 10,000 at C band. It is further observed that the coupling to such a resonator can be tight enough to allow a